4.2. Categorical Sentences

We now shift gears, turning to a distinct form of logic with an equally venerable heritage. While the logic of the last two chapters traces back to the work of the ancient Stoic logicians, the type of formal logic explored here finds its roots in the writings of Aristotle. The guiding idea will be the same as before, however: the validity or invalidity of an argument depends solely on its logical form. What changes here is just the kind of logical form under study.

The following two arguments, for example, strike us as clearly valid.

> All gamblers are people who take risks.
> All people who take risks are people Jack likes to party with.
> (So,) All gamblers are people Jack likes to party with.

> [All surfers are fit.
> No TV critics are fit.
> (So,) No surfers are TV critics.]

Again we see a common formal skeleton underlying both arguments, which we can depict like so (using "G," "H," and "I" as blanks where subject matter goes).

> All G are H
> All H are I
> (So,) All G are I

But note well what remained when the subject matter was stripped away, and what went away. The subject

[Term logic]

1. Categorical Sentences. The Aristotelean tradition

Each categorical sentence begins with a **quantifier** phrase – either “All” or “Some”. If the sentence begins with “All” its quantity is **universal**; it is a **universal sentence**. If the sentence begins with “Some” its quantity is **existential**.

Following the quantifier, a categorical sentence has two plural noun phrases. Each of these is a **term**. The two terms of a categorical sentence are linked by the word “are”. Here are some examples.

> All *men* are *mortal beings*.
> Some *men* are *mortal beings*.

Each term in a categorical sentence also has a **value**[1]. If the term begins with “non-” that term is **negative**; otherwise it’s **positive**.

Positive Terms:	**Negative Terms**:
men	non-men
mortal beings	non-mortal beings

The first term of a categorical sentence (between the quantifier and “are”) is the **subject** term. The second term of the sentence (after “are”) is the **predicate** term.

[1] Something similar to “value” is been traditionally called “quality”. But quality applied to the entire categorical sentence, whereas value here applies to a single term. In this presentation, **sentences do not have values, only terms do**. However, if we restrict ourselves to the four traditional sentence-types discussed below, the value of the sentence is just the value of its **predicate** term.

2. Categorical Sentences and Categorical Syllogisms. A class of arguments (called "**categorical syllogisms**") employ just the four kinds of sentences listed below, said to be in "**categorical form**".

Square of Opposition

(A) "All G are H"
All G are H

(E) "No G are H"
All G are non-H

(I) "Some G are H"
Some G are H

(O) "Some G are non-H"
Some G are non-H

Here "G" is the subject, "H" the predicate.

A **categorical syllogism** is an argument with two premises and one conclusion, each in categorical form. Both the following arguments qualify as categorical syllogisms.

1. All men are mammals
2. All mammals are warm-blooded animals

∴ All men are warm-blooded animals

1. All men are mammals
2. All mammals are non-lizards

∴ Some men are lizards

In the first of these arguments the conclusion follows validly from the premises, whereas in the second the conclusion obviously does not.

3. Rules for Syllogistic Deductions. While various methods suffice to demonstrate that a categorical syllogism is valid, here we develop a test along the lines of our earlier formal deductions – where the conclusion is deduced from the premises through a chain of inferences, each fitting some inference rule.

Such ‘syllogistic deduction’ calls for only two rules: Switching and Linking.

Switching allows the two terms of a categorical sentence to change places: the subject becomes the predicate, the predicate becomes the subject.[2]

To ensure validity of inference, **Switching in univeral sentences** is **restricted**: when terms are switched in a universal sentence, the **value** of each term must be changed. So if a term is negative before being switched, it is positive afterward; if positive before switching, negative after. English examples illustrate.

All **men** are **mortal beings**.

∴ All **non-mortal beings** are **non-men**.

All **lizards** are **non-mammals**.

∴ All **mammals** are **non-lizards**.

[2] This is very similar to what is traditionally called “conversion,” first treated in Aristotle’s **On Interpretation** Chapter 2.

The general pattern for Universal Switching is like so.

Switching (S): Universal Sentences

All ⬟ are ✱
All non-✱ are non-⬟

All non-⬟ are non-✱
All ✱ are ⬟

All ⬟ are non-✱
All ✱ are non-⬟

All non-⬟ are ✱
All non-✱ are ⬟

In Existential sentences Switching can occur without restriction.

Switching (S): Existential Sentences

Some ⬟ are ✱
Some ✱ are ⬟

English examples illustrate.

Some **men** are **doctors**.
∴ Some **doctors** are **men**.

Some **men** are **non-husbands**.
∴ Some **non-husbands** are **men**.

The second rule, **Linking**, derives a new sentence from two previous sentences. The middle sentence – the **linking premise** – has as its terms the **predicates** of the other sentences. Here are two English examples.

All humans are **mortal beings**.
All **mortal beings** are **creatures requiring food**.

∴ All humans are **creatures requiring food**.

Some mammals are **non-finned beings**.
All **non-finned beings** are **non-fish**.

∴ Some mammals are **non-fish**.

Linking (L)

All ⬟ are ❋		Some ⬟ are ❋
All ❋ are ✖	⇐ Linking Premise ⇒	All ❋ are ✖
All ⬟ are ✖		Some ⬟ are ✖

Linking must obey the following two *restrictions*:

- ✓ The **linking premise** must be **universal**.
- ✓ The **conclusion** must have the **same quantity** as **the other** (non-linking) **premise**. (If the other premise is universal, the conclusion must universal; if the other premise is existential, the conclusion must be existential.)

Skeletal examples illustrate the two deduction rules at work.

Example 1: we demonstrate the validity of the following argument form.

1. All G are H
2. All I are non-H

$\therefore$ All G are non-I

The deduction begins with the premises, and "Get" line for the desired conclusion.

1. All G are H (Premise)
2. All I are non-H (Premise)

Get: All G are non-I

We apply **Switching** to Line (2).

1. All G are H (Premise)
2. All I are non-H (Premise)

Get: All G are non-I

3. All H are non-I (2, S)

Linking leads from Lines (1) and (3) to the desired conclusion – at which point the "Get" line is crossed out.

1. All G are H (Premise)
2. All I are non-H (Premise)

~~**Get**~~: All G are non-I

3. All H are non-I (2, S)
4. All G are non-I (1, 3, L)

Example 2:

1. All G are non-H
2. All I are H

$\therefore$ All G are non-I

1. All G are non-H (Premise)
2. All I are H (Premise)

——————(~~Get~~: All G are non-I)

3. All non-H are non-I (2, S)
4. All G are non-I (1, 3, L)

For each of the following argument forms, the conclusion can be deduced from the premises using just Switching and/or Linking. (The first four require only Linking.)

1. All G are H . All H are I ∴ All G are I
2. All G are H . All H are non-I ∴ All G are non-I
3. Some G are H . All H are I ∴ Some G are I
4. Some G are H . All H are non-I ∴ Some G are non-I
5. Some G are H . All I are non-H ∴ Some G are non-I
6. Some G are non-H . All I are H ∴ Some G are non-I
7. All H are G . Some H are I ∴ Some G are I
8. Some H are G . All H are I ∴ Some G are I
9. All H are G . Some H are non-I ∴ Some G are non-I
10. Some H are G . All H are non-I ∴ Some G are non-I
11. All H are non-G . All I are H ∴ All G are non-I
12. All H are G . Some I are H ∴ Some G are I
13. Some H are G . All I are non-H ∴ Some G are non-I

4. Arguments Requiring Existence Assumptions. Some arguments in syllogistic form are not valid as stated, but can be made valid by adding an existence premise: an additional premise claiming existence for a certain type of thing. For example, the following argument is not valid.

1. All cyclopses are one-eyed giants
2. All one-eyed giants are living beings

∴ Some cyclopses are one-eyed giants

Recall that "some" is read as: there exists at least one. So while both premises may be true, it is still false that there exists at least one cyclops which is a living being – for in fact there exist no such mythical creatures. The actual world is a validity counterexample for this argument.

However, the conclusion would follow if we add the third premise that "There exist some cyclopses." And that existence assumption can be framed in categorical form as follows.[3]

"There exist some G": Some G are G .

Here we make the essential claim – that there exists something which is G – and then add, repetitively, that this thing is G. The second adds no new information; but the repetition does no harm, and allows the sentence to fit into categorical form.

The following arguments are valid with the added existence assumption (in brackets).

1. All G are H . All H are I . [There are G] ∴ Some G are I
2. All G are H . All H are non-I . [There are G] ∴ Some G are non-I
3. All G are H . All I are non-H . [There are G] ∴ Some G are non-I
4. All H are non-G . All I are H . [There are G] ∴ Some G are non-I
5. All H are G . All H are I . [There are H] ∴ Some G are I
6. All H are G . All I are non-H . [There are H] ∴ Some G are non-I
7. All H are G . All H are non-I . [There are H] ∴ Some G are non-I
8. All H are G . All I are H . [There are I] ∴ Some G are I

[3] This is the ordinary 'I' form, but with the same term serving as subject and predicate. Our statement of sentences in categorical form placed no restriction on subject and predicate that would prevent this; but to those would not recognize this as orthodox categorical form it will mark a slight relaxation of what qualifies as a sentence in categorical form.